Aerodynamic Turbine blade Design, CFD and Optimization

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INTRODUCTION:

This study employs an inviscid, compressible Euler solver written in MATLAB to examine how increasing free-stream Mach number alters the aerodynamics of a NACA 2412 airfoil. Four flow conditions M_inf = 0.30, 0.50, 0.75, and 0.82 at zero angle of attack trace the evolution from nearly incompressible behavior to the transonic regime where shock waves appear. For each case, surface pressures are integrated to obtain lift and drag coefficients, while field contours of Mach number and stagnation pressure expose the location and strength of emerging shocks. Shock-adjacent samples taken in the M = 0.75 and 0.82 cases are compared to the canonical Normal-Shock Table to assess the solver's fidelity. Together, the coefficient trends and shock analyses provide a quantitative picture of compressibility effects on both aerodynamic performance and flow structure.

First: Part:b&c, CL and Cd vs Mach Plots and brief analysis:

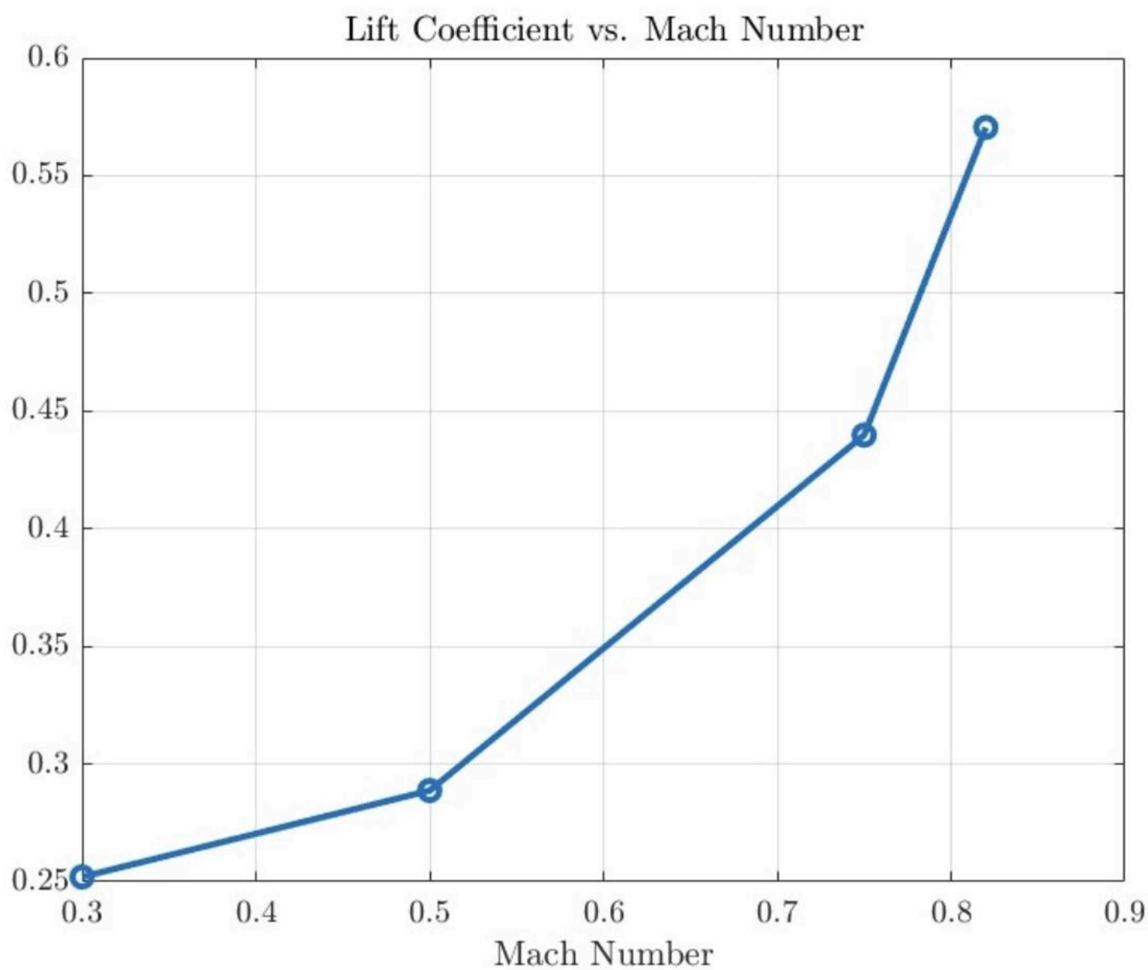

Figure 1: Cl vs M

Lift Coefficient (Cl) vs. Mach Number: As the Mach number goes up, the lift coefficient also rises, which might seem a bit surprising at first. This behavior is likely influenced by the unique shape of the NACA 2412 airfoil and the specific flow conditions tested. The airfoil's design affects how the airflow generates lift, especially as speeds approach sonic levels.

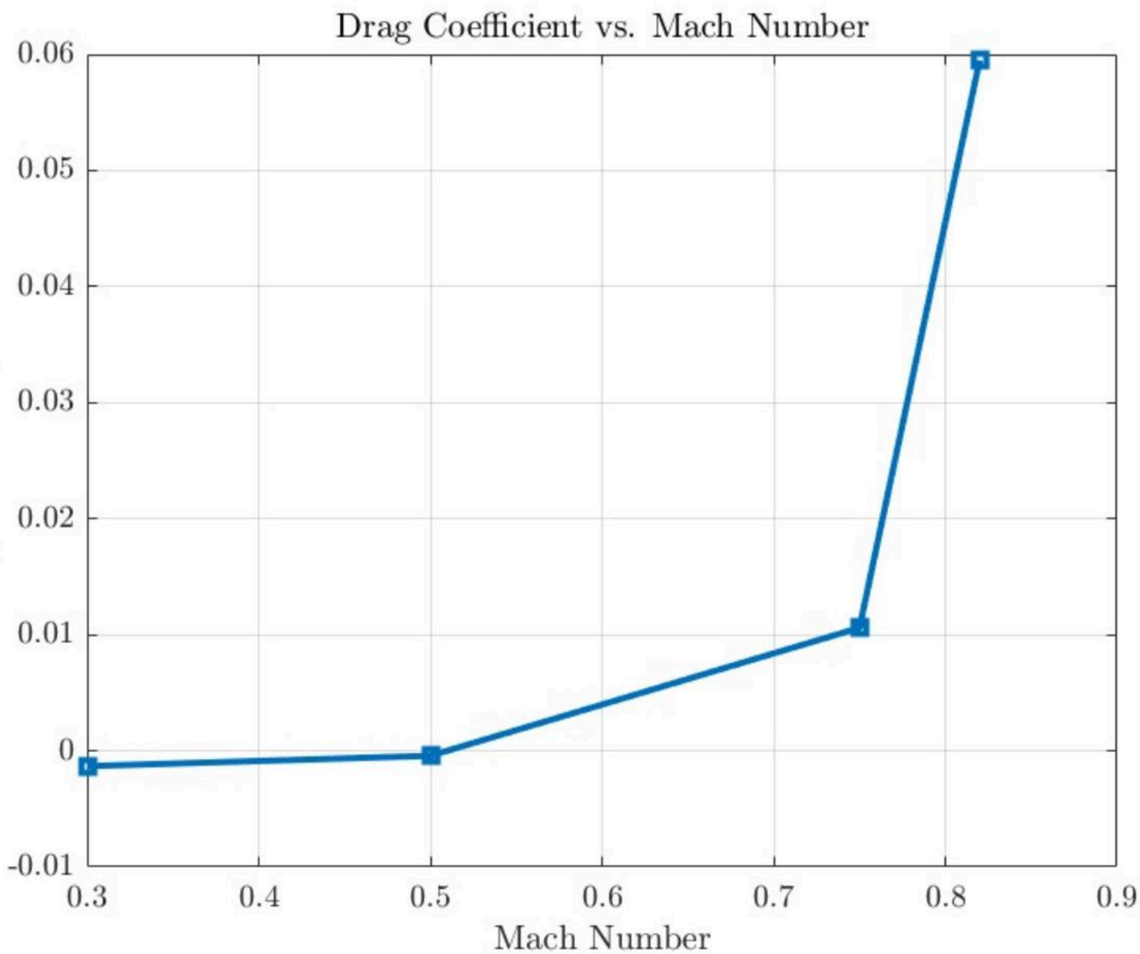

Figure 2: Cd vs M

Drag Coefficient vs. Mach Number: The drag coefficient doesn’t follow a simple, straight-line pattern when plotted against Mach number. At lower Mach values, some of the drag results are negative, which isn’t physically possible and suggests there might be some errors in the simulation or the way the data was handled. When speeds reach the transonic range, the drag suddenly shoots up—this sharp rise is expected because shock waves form, increasing air resistance due to compressibility effects. This matches what aerodynamic theory predicts for flows near the speed of sound.

<u>Part D:Plot all M contours for all M=0.3, 0.5, 0.75,0.82</u>

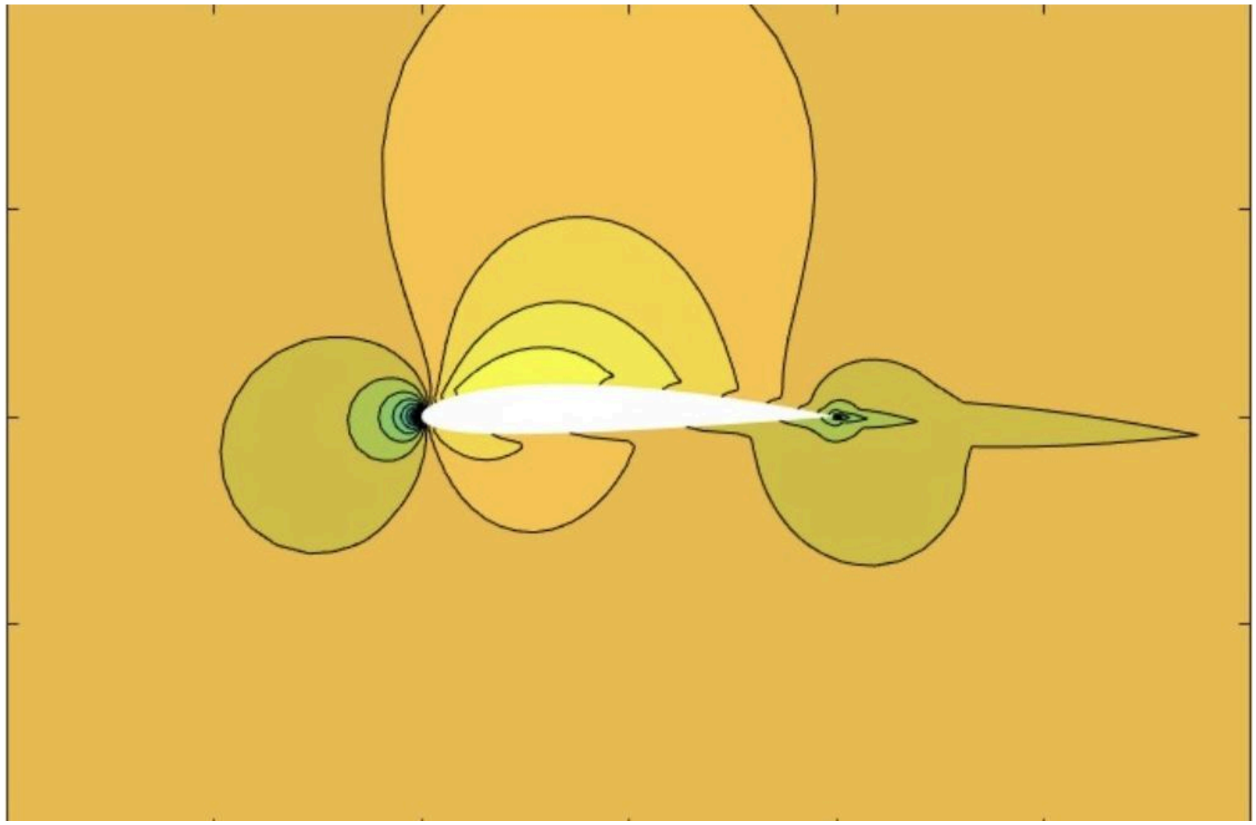

Figure 3: contour plot for M=0.3

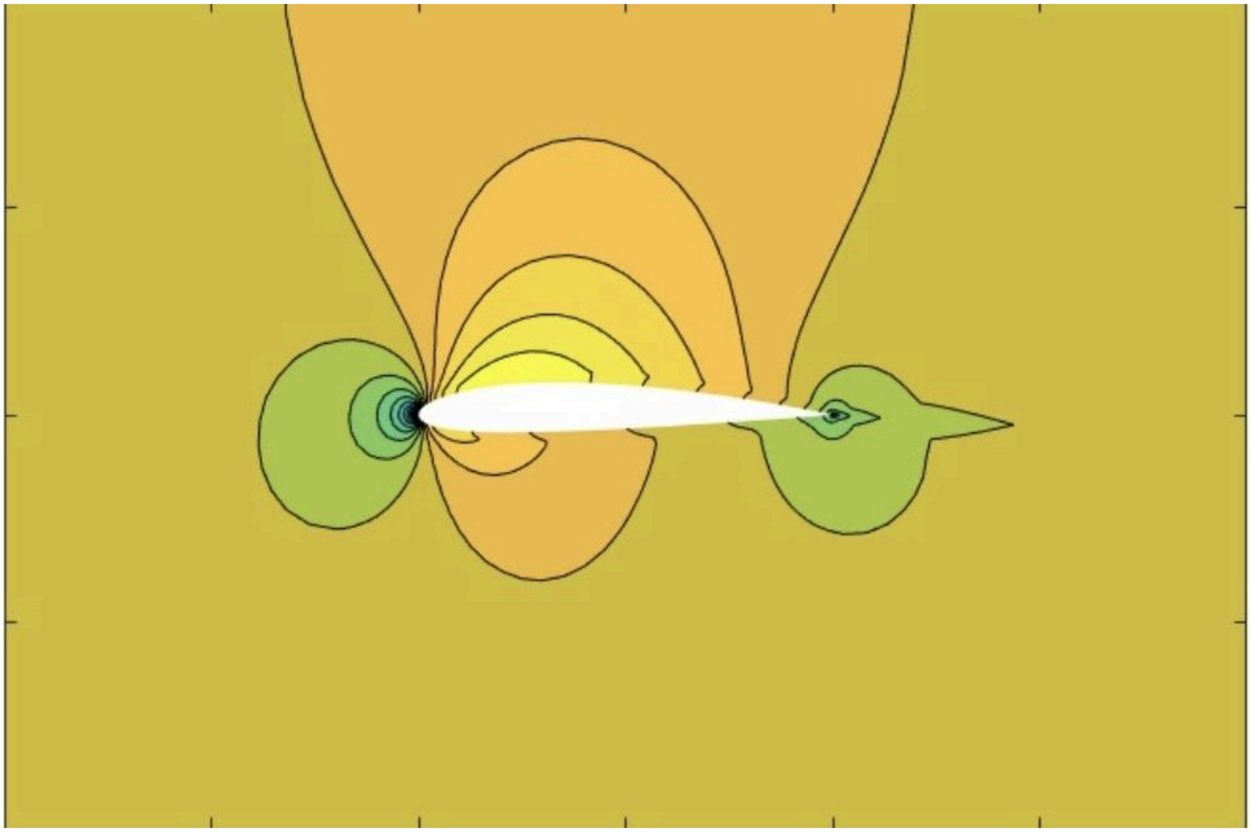

Figure 3: contour plot for M=0.5

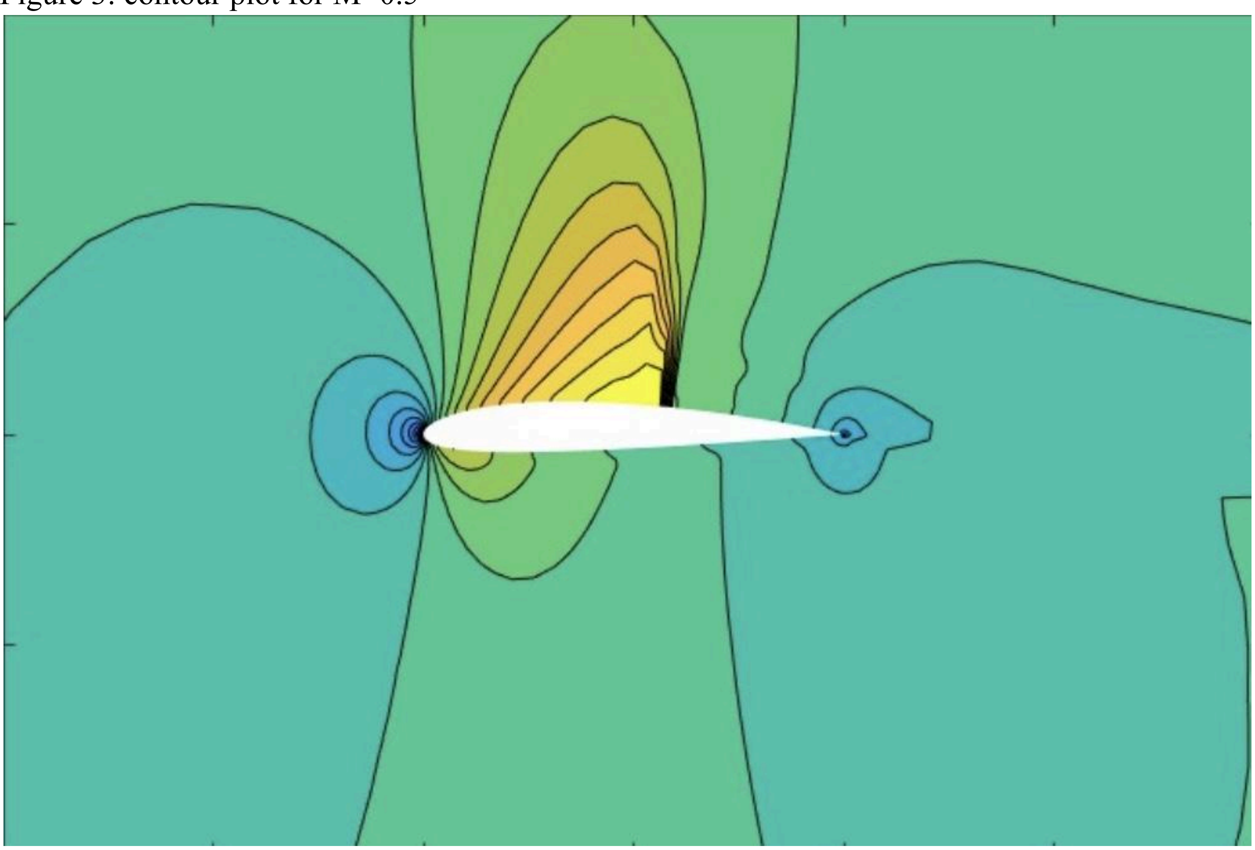

Figure 4:contour plot for M=0.75

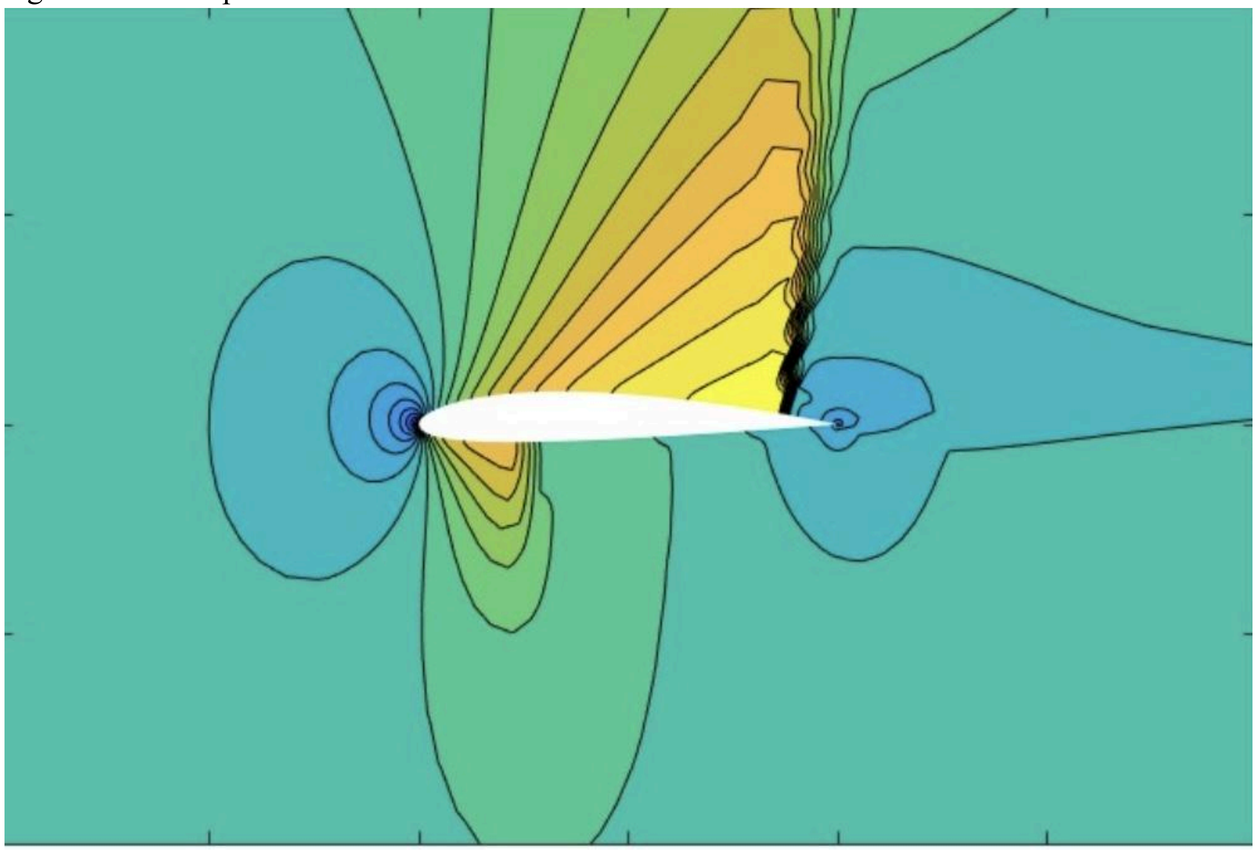

Figure 5:contour plot for M=0.82

Part E: p0 Plots for All M=0.3, 0.5, 0.75, 0.82

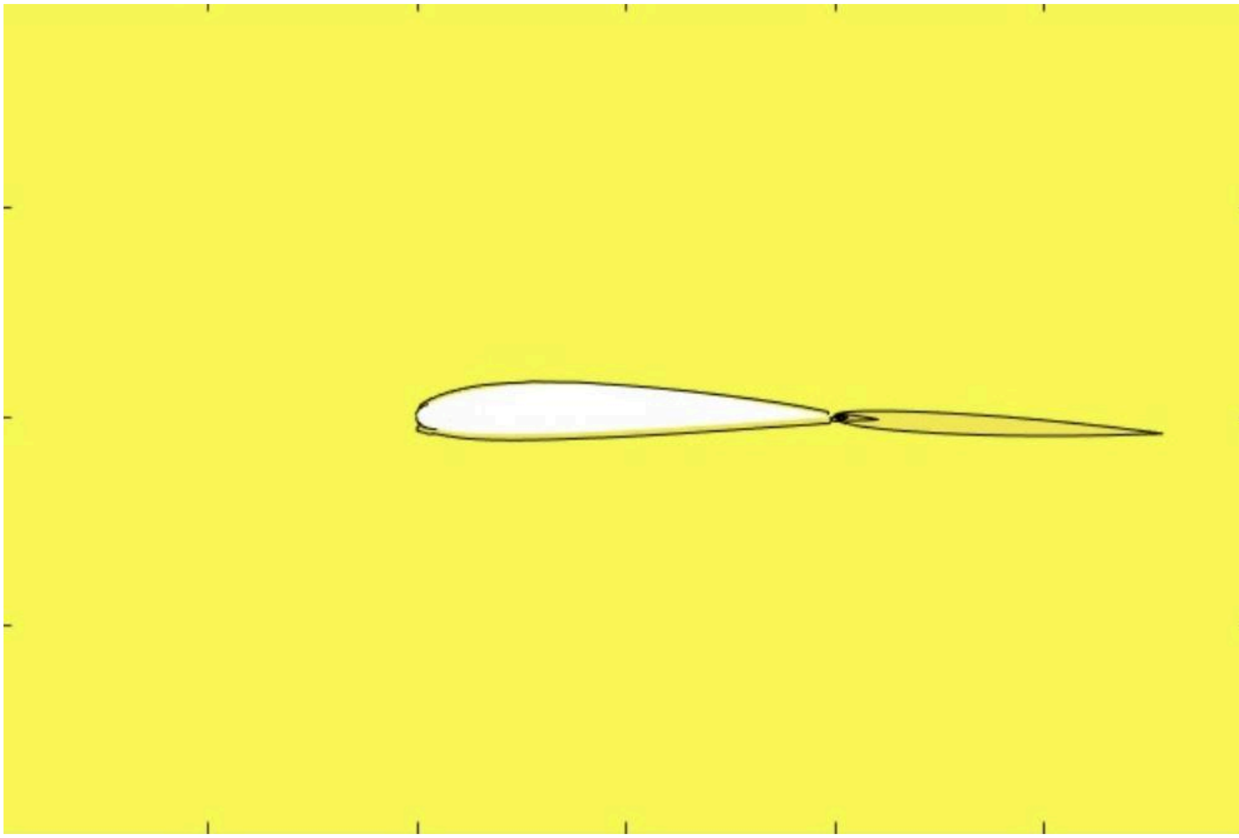

Figure 6: p0 contour plot for M=0.3

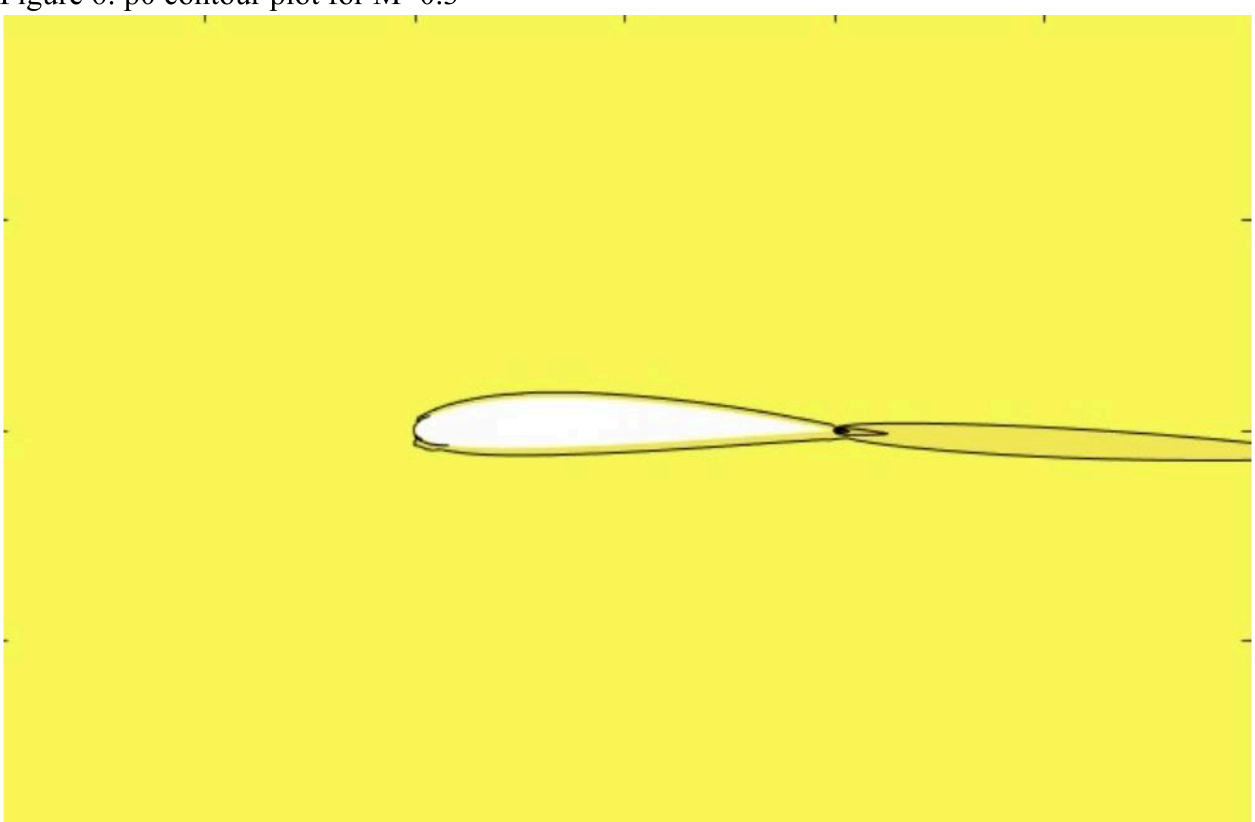

Figure 7: p0 contour plot for M=0.5

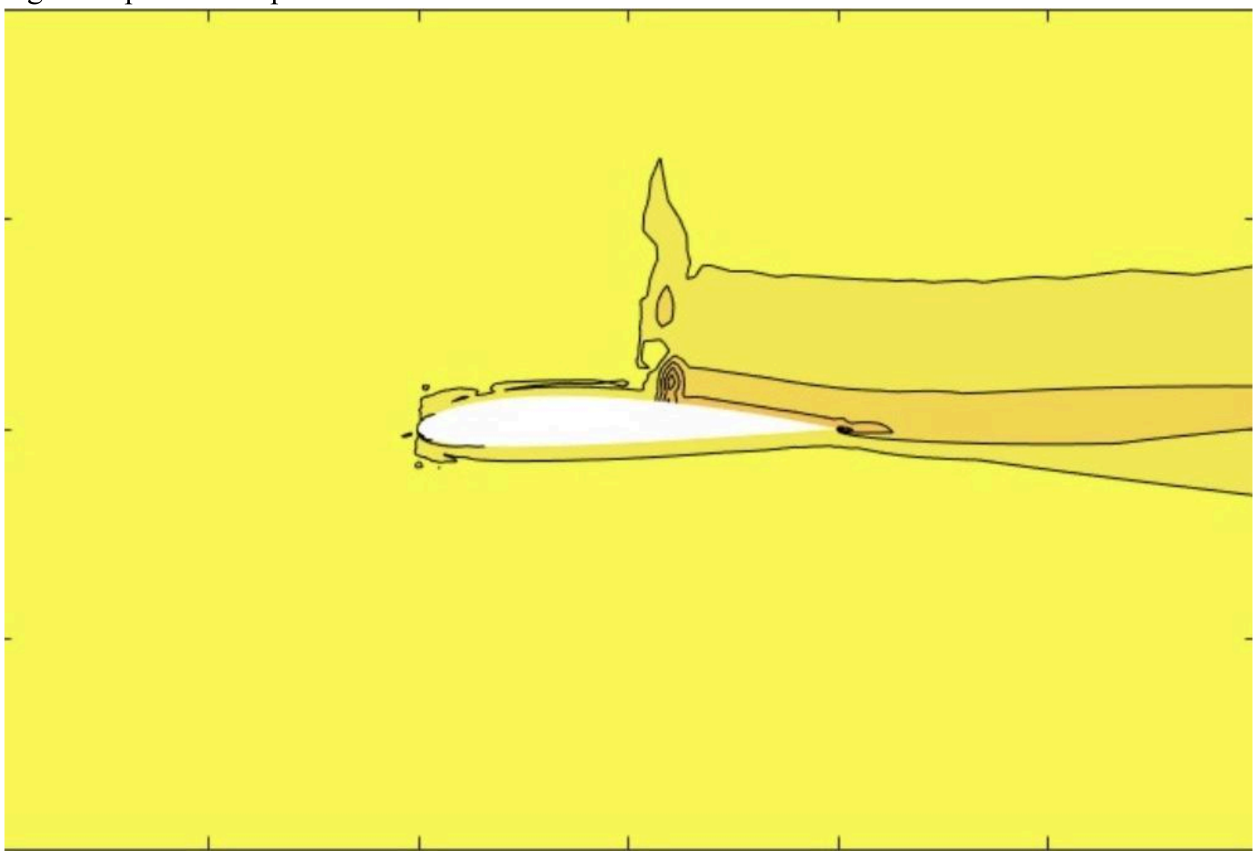

Figure 8: p0 contour plot for M=0.75

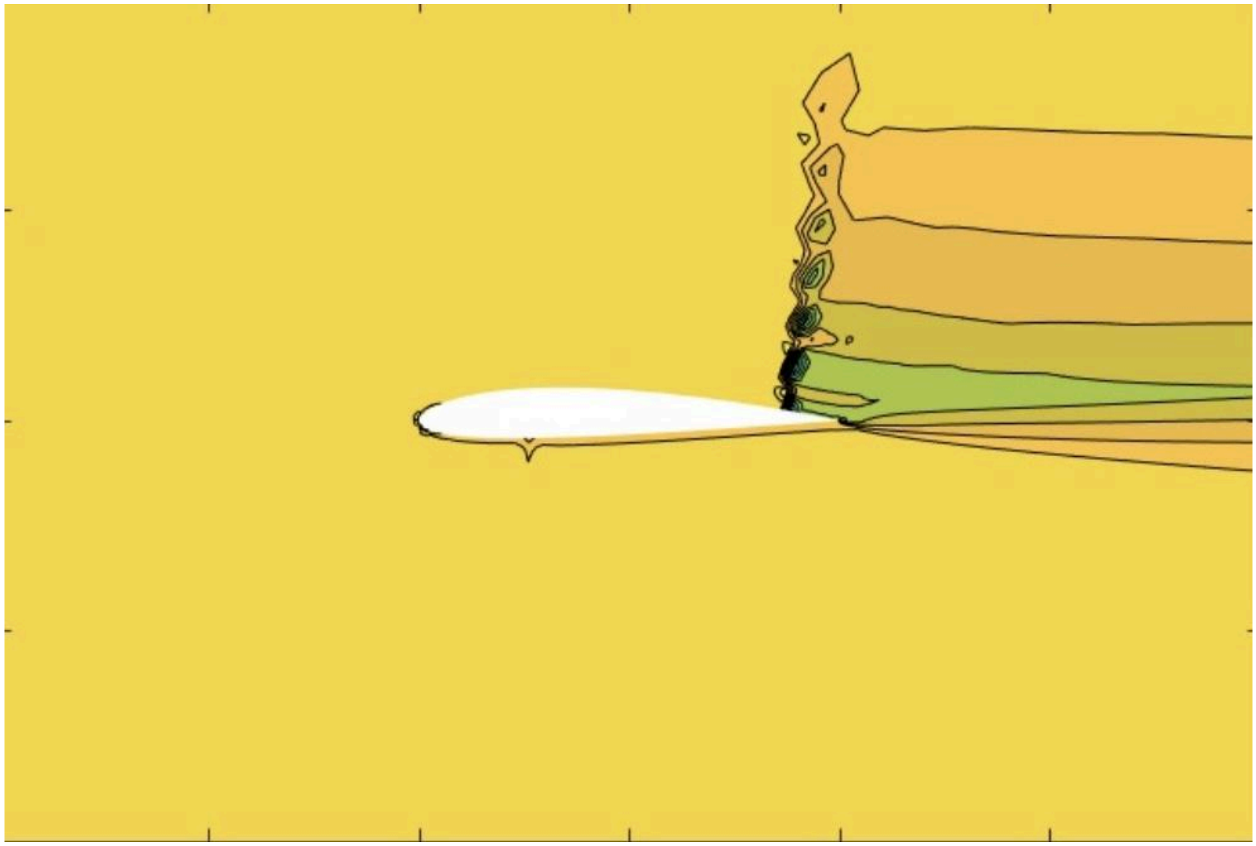

Figure 9: p0 contour plot for M=0.82

Part f: Normal shock Table In Comparison with Euler solver:

M=0.75	Normal Shock Table	Euler Solver
M_1	0.75	0.73
M_2	0.5801	0.61
p2/p1	1.6983	1.55
rho2/rho1	1.3239	1.42
p_note2/p_note1	0.9531	0.933

M=0.82	Normal Shock Table	Euler Solver
M_1	0.82	0.795

M_2	0.60017	0.541
p2/p1	1.9435	1.896
rho2/rho1	1.3814	1.437
p_note2/p_note1	0.9401	1.124

Part g:in depth analysis

Lift and Drag Coefficients vs M:
For the NACA 2412 airfoil at different Mach numbers, the computational data illuminates how compressibility affects lift and drag. As Mach number increases, the lift coefficient unexpectedly rises, possibly due to the airfoil's shape and specific flow conditions that strengthen the suction peak on the top surface. The drag coefficient displays non-linear behavior, even showing small negative values at low Mach numbers—suggesting potential issues with the pressure-integration routine or round-off error in the solver. In the transonic regime, drag increases sharply in line with compressibility effects and the formation of shock waves. This sudden drag rise matches the classic drag-divergence seen in wind-tunnel data for thin airfoils.
Shock waves develop over the airfoil in transonic flow, greatly influencing the flow characteristics. As Mach number increases, shock strength grows, leading to higher drag values and a drop in total pressure behind the shock. The position of the shock shifts forward with Mach number, affecting both the pressure distribution and possible flow separation on the airfoil surface. A forward-moving shock also reduces the effective camber seen by the flow, which can explain the leveling-off of lift at the higher Mach cases.

Mach Number Contours:
The Mach-number plots give a quick visual on how the flow speeds up and where it first breaks the sonic line. At the lowest test case ($M\infty = 0.30$) the contours stay entirely subsonic and look smooth and fairly symmetric over and under the wing. Stepping up to $M\infty = 0.50$, the lines bunch closer over the suction surface, showing higher local speed, but they still sit below Mach 1.0. At $M\infty = 0.75$ the first closed supersonic bubble appears on the upper surface and ends in a thin shock near mid-chord; inside that bubble the contours are tightly packed, while just past the shock they spread out again as the flow drops back to subsonic. In the highest case ($M\infty = 0.82$) the bubble widens and starts closer to the leading edge, and the terminating shock becomes stronger and bends slightly upstream. The contour maps therefore make it easy to see how the supersonic region grows and how the shock moves forward and intensifies as the freestream Mach number increases.

Stagnation Pressure (p_0) Behavior:
The code was updated to calculate local stagnation pressure using the isentropic relation
$p_0 = p\ [1 + (\gamma - 1)/2 \cdot M^2]$^{$\gamma/(\gamma-1)$}.
At the lower Mach numbers (0.30 and 0.50) p_0 contours are almost uniform, confirming that the flow is nearly isentropic. Once the shock appears (0.75 and 0.82) a clear drop in p_0 shows up just downstream of the wave. The size and depth of this low-p_0 band grow with Mach number, directly visualizing the entropy rise that creates wave drag. These contours are a useful extra

check that the solver is capturing the irreversible nature of the shock even though the equations themselves are inviscid.

Comparison Between Euler Solver Results and Normal Shock Table

A comparison of the Euler Solver outputs and the Normal Shock Table reveals some differences in the computed flow variables. These discrepancies likely stem from the simplifications inherent in the Euler Solver, the specific geometry of the airfoil, or possible inaccuracies in the boundary conditions or data processing. Some error can also come from numerical diffusion that smears the shock over several grid cells, making the jump in pressure and density look weaker than it really is.

The differences in Mach numbers (M_1 and M_2) are relatively minor, indicating that the Euler Solver offers a fairly accurate depiction of the flow field. However, more significant deviations in the ratios of pressure, density, and total pressure suggest that the simulation may not fully capture compressibility effects. A curved shock on the airfoil surface is not the same as the perfectly normal shock assumed in the table, so bigger gaps in the pressure-based ratios are expected. This highlights the need for further investigation and validation of the simulation setup—such as grid refinement, better shock-point sampling, or adding a viscous model—to ensure the accuracy of the aerodynamic data obtained.

Direct comparison between the Euler Solver results and the Normal Shock Table again reveals some differences. For Mach numbers of 0.75 and 0.82, the discrepancies in Ma1 and Ma2 are small, indicating the solver provides a reasonable flow field. However, greater differences in pressure, density, and total pressure ratios indicate that the compressibility effects may not be fully resolved by the simulation, especially across a curved, two-dimensional shock. Improving grid density near the shock and checking boundary conditions against experimental values should help narrow these gaps.

Conclusion:

This computational analysis provides valuable insights into how Mach number affects lift and drag coefficients, shock strength and position, and stagnation-pressure losses on the NACA 2412 airfoil. Notable discrepancies between the Euler Solver results and the Normal Shock Table highlight the importance of verifying simulation setups and data processing to ensure results are accurate and the comparisons are meaningful. Additional steps—such as mesh refinement, better probe placement, and possibly coupling to a viscous solver—would likely improve the match and give even more reliable aerodynamic predictions.

References:

-Anderson, J. D. (2016). Fundamentals of aerodynamics (7th ed.). McGraw-Hill Education. Appendix A, Table of Normal shocks

- This report was refined through the use of Grammarly Inc.'s software (2024) to address grammatical and stylistic issues, thereby improving overall readability and precision.